# GPU-based Adaptive Surface Reconstruction for Real-time SPH Fluids (Supplemental Material) 

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In this supplemental material, we describe an idea on how to determine the shape of cracks and show all the possible crack patterns used in the crack filling algorithm of our adaptive surface reconstruction method. All crack patterns are stored in the pattern array described in Section 4.3 of our paper.

## 1 DETERMINING CRACK SHAPES

We determine the shape of a crack in four steps as shown in Figure 1. In step 1, the edge vertices are all generated as a template. Here the upper-left small face as indicated by the red box is an ambiguous face that has four edge vertices and their connections are ambiguous. We use the asymptotic decider method proposed by Nielson and Hamann [Nielson91] to resolve the connection problem. Here we just assume the implicit value sign of the intersection point of the asymptotes, which is the red vertex in the red box illustrated in Figure 1, as positive. In step 2, we separate the common face into the coarse side and fine side. The upper face is the coarse side. The two edge vertices generated are colored in purple. The lower face is the fine side. With the upper face, we assume that fluid flows inside perpendicularly to the rectangular plane, and enters the cross section between the fluid and rectangular plane shaded in light blue in the figure. With the lower face, we assume that fluid flows outside the rectangular plane, thus exiting the cross section shaded in light blue. In step 3, we can see that some parts (shaded in light blue) counteract with each other while others do not (shaded in dark blue). In step 4, the parts that do not counteract each other are defined as cracks (shaded in light yellow).

## 2 DEFINING ALL POSSIBLE CRACK PATTERNS

In order to define all the possible crack patterns, we should analyze all the possible conditions of edge vertices occurring on the level- 2 side of the common face. There are twelve edges in a level- 2 cell face with indices ranging from 0 to 11 . Each edge will potentially generate an edge vertex so that there are $2^{12}$ conditions in total. We classify the edge vertices into two groups: points generated in inner edges with indices $2,3,8$ and


Figure 1: Defining crack shape in four steps.

9 as shown in Figure 2; points generated at outer edges with the other indices. The cracks can also be classified into two groups: Cracks with and without inner edge vertices. Since a crack can at most have four inner edge vertices, we can further classify the cracks into five groups:

Group 1: Cracks with one inner edge vertices.
Group 2: Cracks with two inner edge vertices.
Group 3: Cracks with three inner edge vertices.
Group 4: Cracks with four inner edge vertices.

Group 5: Cracks with no inner edge vertices.


Figure 2: Edge indices of level-2 cell. Edges with indices $2,3,8$ and 9 are inner edges. The others are outer edges.

### 2.1 Analyzing Patterns of Group 1

Here we analyze all the possible crack patterns of Group 1. The other four groups can be analyzed similarly. The patterns shown here are unique in terms of rotation and reflection. Here we take the iso-value to be exactly zero where an edge vertex is generated between two edge vertices with positive and negative implicit function values. We also assume that the implicit function values inside the iso-surface are positive and those outside the iso-surface are negative. We obtain all the patterns of Group 1 as shown in Figure 4.
Because there is one inner edge vertex, the implicit value signs of the five vertices belonging to the inner edges are known. What we do not know are the signs of the other four vertices which are circled in purple as shown on the left side of Figure 4. There are five conditions for four signs: all negative, one positive, two positive, three positive and all positive. All the patterns belonging to each condition are shown on the right side of Figure 4. The green edge vertices form the cracks and the green line segments inside each crack show the triangulation that is used to fill it. However, some of the edge vertices do not generate a crack like the part shown in red circle in Figure 4. That is because the two edge vertices generated from two sides coincide with each other as shown in Figure 3.


Figure 3: Example showing that no crack occurs on common face between two cells of different levels. Here, points A and B are generated coincidently on each side of the common face, resulting in no cracks being formed.

### 2.2 Analyzing Patterns of Group 2

As analyzed above, we can obtain all the possible crack patterns of Group 2 similarly. However, for Group 2 where two inner edge vertices exist, the topology of the two points has two conditions which are shown in Figure 5 and Figure 6, respectively. For some patterns generated from templates containing ambiguously tessellated faces where four edge vertices exist as enclosed by the red ellipse in Figure 5, we use thick blue lines to separate all the possible patterns generated from the same template.

In our crack filling approach, we just triangulate the cracks to fill them. For most of the patterns, this method
does not pose any problem. However, for very few patterns as those enclosed by the purple ellipse in Figure 5, the crack cannot be perfectly triangulated because there exists a crack vertex (vertex p) that does not belong to the edge vertices. An accurate way of dealing with this problem is to calculate the exact coordinate of P by calculating the intersection point of line segments $A B$ and CD and triangulating the corresponding crack part using triangles ACP and BDP. But we choose a simple way in our implementation by just stretching the crack part to the existing edge vertices. In our method, we replace triangles APC and BPD with triangles ABC and BCD. This method is simple compared to the accurate one but it may cause topology errors by generating nonmanifold surfaces, for triangle BCP may be perpendicular to the resulting surface mesh. But in our implementation, this result is good enough to neglect that artifact.

### 2.3 Patterns of the Other Groups

The patterns of the other three groups are shown in Figure 7, Figure 8, and Figure 9.

## 3 REFERENCES

[Nielson91] Gregory M. Nielson and Bernd Hamann. The asymptotic decider: resolving the ambiguity in marching cubes. In Proceedings of the 2nd IEEE Conference on Visualization '91 (VIS '91), 83-91 (1991).


Figure 4: All crack patterns of Group 1.


Figure 5: All crack patterns of Group 2: First condition.


Figure 6: All crack patterns of Group 2: Second condition.


Figure 7: All crack patterns of Group 3.


Figure 8: All crack patterns of Group 4.


Figure 9: All crack patterns of Group 5.

